International Journal of Theoretical Physics, Vol. 19, No. 8, 1980

Nonstationary Quantum Mechanics. I. Time-Irreversible Noninstantaneous Self-Reduction to a Stationary State of Some Quantal Wave Packets

Nickola Stefanov Todorov

Low Temperature Department of the Institute of Solid State Physics of the Bulgarian Academy of Sciences, Boulevard Lenin 72, Sofia 11184, Bulgaria

Received May 19, 1978

It is shown that the formalism of quantum theory needs modification in the case of potential fields swiftly varying with time. The necessity of a time-irreversible "master equation" for such cases is discussed.

The underlying idea is that any (sub)system will undergo a spontaneous transition to a state of definite *energy* in the process of separating spatially from the rest of the "universe," assuming the "universe" is isolated and has a definite energy. This requires what might be termed a "pragmatic" interpretation of the wave function: If a composite, separated system is represented by a linear superposition of product states, we may say that the actual state of the composite system is represented by some particular component of the superposition *for the purposes of statistical inferences relevant to each subsystem alone*, but the entire superposition—and not the corresponding mixture of the product components—must be used to compute the statistics of correlations. The considerations are illustrated with thought experiments which are real enough to make the application of the usual quantum mechanical formalism possible. Cases of disagreement between conventional theory and experiment in the field of interest are indicated.

1. INTRODUCTION

In 1935 Einstein, Podolsky, and Rosen published a famous theorem (Einstein et al., 1935) known since then as the EPR paradox. This inappropriate name for the said theorem was given, most probably, because of the previous attempts of Einstein to find a paradox in quantum theory, the statistical character of which he profoundly disliked. Thus the EPR theorem was misunderstood or misinterpreted from the very beginning, which

0020-7748/80/0800-0605$03.00/0

is clearly seen in the series of early refutations: Bohr (1935) thought that it was a criticism of the mathematical apparatus of quantum mechanics; in the same line of reasoning Furry (1936) interpreted it as an assertion about reduction of the initial wave packet, while Mandel'shtam (1950) explained to his students "the error of Einstein" by stressing that the wave function ψ refers to ensembles of identically prepared systems and not to a single quantum system, i.e., exactly the idea which EPR wished to convey. The steady flow of refutations of the EPR theorem continues to the present day and only lately some authors (Ballentine, 1970; Ross-Bonney, 1975) came to the realization that it is a just criticism of the idea that the state vector provides a complete description of an individual system. This was quite patiently explained by Einstein himself (Einstein, 1948, 1953), who pointed out, besides, that in such a case no proper transition exists from the statistical quantum theory to classical mechanics, which is concerned with the description of the state of motion of individual systems. So, it is not unnatural to assume that hidden-variable (HV) theories exist which make possible the description of the evolution of individual systems.

The latter problem has evoked extensive debates in the literature. The earliest impossibility proof belongs to von Neumann (1932), whose mathematical genius is so impressive that for quite a long period of time his cited book was considered to provide a perfect axiomatization of quantum theory, thus making it a theory devoid of difficulties from the point of view of logic. It was again only recently realized by some authors (Bell, 1966; Bohm and Bub, 1966; Ballentine, 1970; de Broglie et al., 1976) —that mathematical and physical rigor are not the same thing and that one has to be careful in evaluating the formal approaches to quantum mechanics. This applies not only to the impossibility proofs but to von Neumann's theory of quantum measurements as well (Ballentine, 1970; de Broglie et al., 1976) the "telepathic" character of which (as de Broglie et al. put it) is no longer acceptable for a contemporary physicist. (As we shall see in T2[1] a great deal of inconvenience was caused by von Neumann's approach to quantum statistical mechanics as well.)

The present work is not intended to be a review article on the problems mentioned above and we shall not discuss them in detail. A detailed discussion of the impossibility of an efficient impossibility proof will be given in our work T6. Extensive considerations of the subject from a point of view somewhat different from the one accepted in T6 can be found in the cited works. [The essence and some difficulties of the orthodox theory of measurements are discussed by Wigner (1963).] The

[1]We have cited works denoted by T2, T3,..., T7 in this paper. The works T2,..., T5 are, correspondingly, Part II,..., Part V of the present article entitled "Nonstationary Quantum Mechanics." The works T6 and T7 will be submitted for publication.

brief survey of the said problems was necessary owing to the fact that we shall encounter them below in a different line of reasoning. More exactly, we shall see that quantum mechanics is not free of difficulties of logic, which can possibly lead to numerically wrong results in the case of perturbations swiftly varying with time. One of our thought experiments is of the EPR type and its discussion will lead to the necessity of a picture in which some wave packets automatically reduce to states of definite stationary energy, but it is considered from a different standpoint (EPR did not mean a thing like that, as Furry (1936) suspected), which does not refer to the problem of completeness of quantum theory directly. We shall be forced, besides, to take a reasonable (as we hope) attitude about the nature of the correlations in the quantum motion of systems which are spatially separated (isolated) after they have interacted in the past. The possible necessity of an HV theory will be examined likewise. The corresponding considerations will be carried out in a scope sufficient for the aims of the present work.

In T2 we shall apply the ideology developed in the present Part I to the problem of entropy increase with time. In T3 we shall show that in the case of swift variation of potentials with time one does not come to classical momentum distributions (in the classical limit of highly excited states) exactly in those ranges of momentum values where coincidence of the two theories is expected. In T4 and T5 we shall examine the case of slow (adiabatical) variation of potentials with time and shall show that the Schrödinger equation (SE) does not agree with an adiabatical principle in thermodynamics in the case when the energy spectrum of the (sub)system of interest is discrete T4, while in T5 we shall examine the case of a system with a continuous degenerated energy spectrum, pointing out the similarities and the essential differences between the said two cases.

2. A DIFFICULTY OF QUANTUM THEORY IN THE CASE OF PERTURBATIONS SWIFTLY VARYING WITH TIME

In the different impossibility proofs (or, more exactly, assertions—T6) the authors concentrate their attention on the rules of constructing and calculation with the quantum mechanical operators (cf. the reviews of Bell, 1966, and Ballentine, 1970). But these rules do not constitute a complete system of axioms of quantum mechanics. A most important postulate of this theory is the evolution equation for the wave function, i.e., the Schrödinger equation. (We are interested only in nonrelativistic quantum mechanics in all parts T1–T5 of our article.) The application of the SE to the description of the evolution of a quantum system in well-defined external fields has two aspects:

(1) The evolution of our system with time is given by the equation

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=\left[H_{kx}+V_x+U(x,t)\right]\psi(x,t) \tag{2.1}$$

where x denotes the set of all degrees of freedom of the system, H_{kx} is the operator of its kinetic energy, V_x the operator of the interaction between its constituents, and $U(x,t)$ the operator of its potential energy in the external field which may be time dependent.

(2) The evolution of the total system consisting of the system x and the field sources y is given by a wave function $\psi(x,y,t)$, satisfying the equation

$$i\hbar\frac{\partial\psi(x,y,t)}{\partial t}=(H_{kxy}+V_{xy})\psi(x,y,t) \tag{2.2}$$

where xy denotes all the degrees of freedom of the overall (isolated) system.

But it is well known that when the evolution of the overall system is described by a wave function $\psi(x,y,t)$ one has to employ a density matrix $\rho(x,x',t)$ to the description of the evolution of a part x of it and not a wave function $\psi(x,t)$, generally. Thus we have a point here in which the said two aspects of the SE overlap. This point can turn out to be a source of difficulties. The aim of our discussion is to show that this is really so. We shall consider in detail a specific example since, as we hope, maximum clarity can be achieved in this way. Our only assumption will be conservation of energy in the overall system xy.

Our first *Gedankenexperiment* will be of the EPR type with respect to the possibility of making definite conclusions about a part of a system by carrying out measurements on the other part, spatially separated from the former one after interacting with it in the past. But we shall examine this situation from a different point of view, thus showing that EPR could really find a paradox in quantum mechanics if they were seeking one.

Quantum mechanics, presumably, is a theory applicable to all possible kinds of interactions in nature (electromagnetic, nuclear, etc.), so we shall examine a case where two types of interactions are present. But it will be clear that the difficulty which arises in this case is not due to the presence of forces of a different nature since the reader will be able to construct for himself situations where only one type of interaction exists [e.g., electromagnetic interaction in the case of a neutral atom in a state of definite energy, entering or going out of an (ideal) capacitor].

We assume that we have an ionized atom of the following kind. A very heavy particle I of a mass $m_1\to\infty$ and positive electric charge e forms

a nucleus with another particle 2 and positive charge e owing to interaction with an intense attractive short-range force f_{12} of a nonelectromagnetic character. An electron 3 of a negative charge $-e$ is in a state of definite energy E_3 in the field of the nucleus, the average radius $\langle r_3 \rangle$ of its "orbit" being much larger than $\langle r_2 \rangle$ owing to the assumed properties of f_{12} [the zero of the coordinate system is in the point where I is located $(m_1 \to \infty)$]. Let the nucleus be in a state of definite energy E_{12}. The atom is bombarded with electrically neutral particles 4 having a velocity much larger than the average velocity of rotation of 3 about the nucleus and interacting strongly with 2 ($f_{34}=0$, $f_{24}\neq 0$). We are interested in that part of the collisions between 4 and 2 in which 2 leaves the atom "instantaneously," i.e., with a velocity much larger than $\langle v_3 \rangle$ (the fact that such a collision has taken place can be registered in principle by an "electromagnetic" instrument measuring the sudden change of the field of the nucleus; this instrument can be located far from the nucleus. Scattering theory, naturally, has to ensure conservation of energy in the overall system $(E_i = E = E_{12} + E_3 + E_4 = E_f)$ in the cases of interest.

In the said cases we have a well-defined potential $U(\mathbf{r}_3, t)$ for all moments of time so that, according to the prescriptions of quantum mechanics, we can use the postulate expressed by equation (2.1) for the description of the evolution of the state of the electron 3, i.e., to find the wave function $\psi(\mathbf{r}_3, t)$ for moments $t>0$, $t=0$ being the moment of the collision and $\psi(\mathbf{r}_3, t)_{t=0}$ being known. In other words, this postulate says in fact that the above-mentioned device is the only apparatus necessary to conclude that we have a wave function $\psi(\mathbf{r}_3, t)_{t>0}$ of the form

$$\psi(\mathbf{r}_3, t)_{t>0} = \sum_n a_n \exp^{(-iE_{3n}t/\hbar)} \psi'_n(\mathbf{r}_3) \tag{2.3}$$

where $\psi'_n(\mathbf{r}_3)$ are the eigenfunctions of 3 in the new field $(t>0)$ corresponding to energies E_{3n}, n denoting the corresponding complete set of quantum numbers of 3. The constant coefficients a_n are determined in a well-known way:

$$a_n = \int \psi(\mathbf{r}_3, 0) \psi'^*_n(\mathbf{r}_3) d^3\mathbf{r}_3 \tag{2.4}$$

The state $\psi(\mathbf{r}_3, t)_{t>0}$, clearly, is a state of indefinite energy, $|a_n|^2$ giving the probabilities for the corresponding eigenstates according to current axiomatics.

On the other hand, another observer can measure the kinetic energies E_{k2} and E_{k4} of 2 and 4 after the collision (this observer can have the information about the moment of the collision by using the same electromagnetic device). This can be done, for instance, using their ability for

nonelectromagnetic interactions. According to the postulate (2.1) this process has no significance at all for the evolution of the state of motion of 3 since the only factor determining this evolution [according to (2.1)] is $U(\mathbf{r}_3, t)$ and the initial conditions for $\psi(\mathbf{r}_3 t)$ (certainly, the measurement of E_{k2} and E_{k4} is carried out in a remote district where the electromagnetic field of 2 cannot influence U noticeably). But the second observer, who is supposed to know the exact value $E_i = E$ of the total initial energy, will obtain definite values of E_{k2} and E_{k4} in the process of measurement irrespective of whether the two particles leave the atom in states of well-defined individual energies or not, and the only possible conclusion for him will be that the atom is in a state of definite energy $E_{3n} = E - E_{k2} - E_{k4}$ after the collision owing to the necessity of energy conservation. (The measured sum $E_{k2} + E_{k4}$ certainly cannot be arbitrary; it has to be such that its combination with one of the eigenvalues E_{3n} gives exactly $E = E_i = E_f$.)

We have, evidently, a contradictory situation here—the state of the system must be defined objectively and should be the same for all observers, while in our case the first observer who relies on the axiom (2.1) obtains a wave function, (2.3), and the second observer who employs (2.2) comes to a fixed ψ'_n after measurement over the initial incoherent mixture of states $\{\psi'_m\}$ of the electron.

This fact has experimental implications. In a nonstationary variant of description of the collision process which is always possible, at least *in principle* (let us remind the reader that such a variant exactly is employed in quantum field theory; cf. any book on the subject), observer 2 will obtain, eventually, an overall wave function corresponding to a definite energy E. The part of the said wave function which is of interest to us from point of view of a comparison with the inferences of observer 1 in the cases of potential jumps can be written schematically in the form

$$\psi(\mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4, t) = \sum_{n,i} c_{ni} \varphi_{ni}(\mathbf{r}_2, \mathbf{r}_4) \psi'_n(\mathbf{r}_3) e^{-iEt/\hbar} \tag{2.5}$$

where $\varphi_{ni}(\mathbf{r}_2, \mathbf{r}_4)$ are eigenfunctions of $H_{k2} + H_{k4}$ satisfying the requirement $E_{k2,k4,n} + E_{3n} = E$ (the index i takes account of the degeneration of the energy spectrum of particles 2 and 4) and c_{ni} are constant coefficients. The registering of the potential jump by observer 2 makes it possible for him to evaluate the time interval since the moment of the reaction after the elapsing of which he would have to say that the state of 3 is a given specific ψ'_n which participates in the total wave function, (2.5). The said interval is determined with the help of the uncertainty relation $\Delta E \Delta t \sim \hbar$ (see, e.g., Section 112 of Blokhintsev's book (1963) and the discussion in T5). So, the probability of finding 3 in a definite point $\mathbf{r}_3$ after such an

interval of time, according to observer 2, is practically constant with time and is given by the incoherent sum

$$\int |\psi(\mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4, t)|^2 d^3\mathbf{r}_2 d^3\mathbf{r}_4 = \sum_n |b_n|^2 |\psi'_n(\mathbf{r}_3)|^2 \tag{2.6}$$

where $|b_n|^2 = \Sigma_i |c_{ni}|^2$. On the other hand, observer 1 will declare that the coordinate probability density is essentially time dependent and is given by the coherent sum

$$|\psi(\mathbf{r}_3, t)|^2 = \sum_{m,n} a_m^* a_n \psi'_m(\mathbf{r}_3) \psi'_n(\mathbf{r}_3) \exp\left[i(E_{3n} - E_{3m}) t/\hbar \right] \tag{2.7}$$

[See equation (2.3); the fact that $\psi(\mathbf{r}_3, 0)$ is a well-defined quantity in our overall system is demonstrated in Appendix A.]

Consequently, we came to a situation in contradiction to the point of view of the interpretation of the square of the moduli of the wave functions. But the disagreement may go still deeper: according to observer 1 the probability of finding 3 in a state ψ'_n is equal to $|a_n|^2$, while according to observer 2 this probability is equal to $|b_n|^2$. And one cannot expect *a priori* that $|a_n| = |b_n|$ (certainly, the normalization of (2.5) is chosen so that $\Sigma_n |b_n|^2 = \Sigma_n |a_n|^2 = 1$ for the sake of comparison).

Indeed, one comes to the sets of values of $|a_n|$ and $|b_n|$ in essentially different ways. The values of $|b_n|$ are obtained by considering a specific experimentally possible situation and taking into consideration the type of the interactions and the existing conservation laws. The constants a_n are obtained (in our case) using only the fact of the quasi-instantaneous character of inclusion of the perturbation. The two ways of reasoning thus differ radically and one is justified to expect, generally, disagreement, the acceptable axiom being (2.2).

For "continuity" considerations it is clear that the possible disagreement between (2.1) and (2.2) has to exist not for quasi-instantaneous perturbations only. It is impossible to say at present where exactly the limit lies beyond which (2.1) is no longer an acceptable axiom. It is evident, though, that this limit exists. Indeed, the postulate (2.1) is well confirmed by the experiment in the other limiting case [$U(x,t) = U(x)$, $-\infty < t < \infty$]. The same considerations show that (2.1) is acceptable for slowly varying perturbations (with respect to the characteristic times of the specific problem). A detailed consideration of such processes is given in T4 and T5.

Obviously, it would be very important to verify experimentally that (2.1) and (2.2) disagree. It is as obvious that one has to turn to experiments in which quasi-instantaneous jumps of the potentials actually exist. Jumps of this sort usually take place in the cases of β decay of nuclei since the β

particles are much faster than the atomic electrons in the majority of events. The only (inessential) complication in this case is the necessity to take into consideration the recoil of the atom (molecule). In the article by Wolniewicz (1965) is discussed exactly a case of this sort. He has computed carefully, using (2.1), the disintegration rates of the HeH^+ molecules which are formed after the β decay of HT. His result–at least 17.8%–is in gross disagreement with experiment ($6.8 \pm 1.9\%$).

The above striking disagreement between theory and experiment is in accord with the gap between the logic of (2.1) and (2.2) in the case of interest. And this is not an isolated phenomenon which can be attributed to some error of a trivial character. The discussion of Feinberg (1965) is evidence of that. This author considers the experimental results of Suzor (1960) and Spighel and Suzor (1962) for ionization probabilities after a β decay of some nuclei. One encounters here again marked disagreement between experiment and the theoretical framework established by (2.1). Feinberg comes to some *qualitative* improvements of the initial theory by taking into consideration the ionization probabilities due to the possibility of direct collisions between the atomic electrons and the fast β particles which leave the nucleus. But this does not remove the problems.

Denote with $E_1 = mc^2 + E_\beta$ the energy of the β particle and with $E_2 = mc^2 + \epsilon$ the energy of the electron which leaves the atom in the process of ionization. Then, as Feinberg points out, we have the following.

(i) In the case of P^{32} the theoretical probability of ionization value W is 2–3 times larger than the experimental one in the region $1.2\ \text{keV} < \epsilon < 2.5\ \text{keV}$, the theoretical curve, though, having a correct form. But in the region $6\ \text{keV} < \epsilon < 12\ \text{keV}$ even this quality of the curve disappears and the decrease of W_{exp} with the increase of E_β for $E_\beta \gtrsim mc^2$ is inexplicable from the theoretical point of view.

(ii) After integrating over the entire spectrum of the β particles one comes to the dependence of the ionization probabilities on ϵ. Some of Suzor's results (1960) for Na^{22} belong to that range of ϵ values ($\epsilon \gg I_k$, I_k being the ionization energy of the K shell) in which theory is expected to give exact quantitative results. But here exactly theory departs most radically from experiment: the theoretically predicted decrease of W with ϵ is given by an ϵ^{-2} law, while the experimentally observed law is ϵ^{-1} which leads to disagreement between theory and experiment amounting to an order of magnitude for large ϵ.

These results are quite significant. Further experimental and theoretical work in the field of fastly varying potentials is necessary so that a complete picture of the exact status may be drawn. As for us, we shall continue in this paper and those to follow the discussion of the logical foundations of the nonstationary problem.

3. REINTERPRETATION OF THE WAVE FUNCTION OF A SYSTEM CONSISTING OF SPATIALLY SEPARATED SUBSYSTEMS

The above discussion of the incompatibility of the postulates (2.1) and (2.2) in the general case comes in conflict with some current interpretations of wave functions of the type given by equation (2.5). The present section is dedicated to the clarification of the problem about the actual information given by such wave functions.

Wave functions of the type (2.5) show that the different parts of a quantum system are described by a density matrix. In our specific *Gedankenexperiment* this means that the subsystem formed by the electron 3 in the field of the nucleus has undergone a transition from an initial state of definite energy E_3, described by a wave function $\psi_i(\mathbf{r}_3)$, to a final state, described by a density matrix $\rho(\mathbf{r}_3, \mathbf{r}'_3)$ (from the point of view of observer 2). We shall give here an interpretation of this process by clarifying the physical meaning of the correlations in the motion of subsystems, expressed by wave functions of the type (2.5). In order to evade inessential complications we are going to examine first the case of an overall system, consisting of two subsystems with degrees of freedom correspondingly x, y, the eigenstates $\psi_i(x)$, $\varphi_k(y)$ of which (after the separation) are supposed to be nondegenerate and orthonormal [$\int \psi^*_{i_1}(x)\psi_{i_2}(x)dx = \delta_{i_1 i_2}$, and so on]. Conservation of energy is assumed in the process of separation.

The total wave function

$$\phi(x,y) = \sum_i a_i \psi_i(x)\varphi_{k_i}(y), \qquad E_{ix} + E_{k_i y} = E \tag{3.1}$$

reduces to an incoherent mixture of states $\psi_i(x)$ with corresponding probabilities $|a_i|^2$ for an observer connected with subsystem x (and, correspondingly, for an observer connected with y). This is true, evidently, for all possible measurements carried out over x (or y) since, for instance,

$$\langle \hat{O}_x \rangle = \int \phi^*(x,y)\hat{O}_x\phi(x,y)dxdy = \sum_i |a_i|^2 \int \psi_i^*(x)\hat{O}_x\psi_i(x)dx \tag{3.2}$$

where $\hat{O}_x$ is the operator of an arbitrary physical quantity of subsystem x. Consequently, the correlations of the motion of x and y, do not exist for observer x. These correlations do not have any influence on the motion of x (or y) which is expressed by the fact that the said measurements yield results which would be the same if the universe consisted only of noninteracting copies of subsystem x (or y) with relative frequencies given by $|a_i|^2$.

On the other hand, a simultaneous measurement of the coordinates of subsystems x and y will give a result corresponding to a density of probability $|\phi(x,y)|^2$ and not to the simpler expression $\Sigma|a_i|^2|\psi_i|^2|\varphi_{k_i}|^2$, and it seems that we do not have the right to say that subsystem x is *really* in a state of motion $\psi_i(x)$ and y in a state $\varphi_{k_i}(y)$ (with a probability equal to $|a_i|^2$). Facts of this sort have generated the idea of a specific interaction at arbitrary distances due to the existence of a "quantum potential" Q which has a definite (though different) value for any possible state $\phi(x,y)$ (see, e.g., the work of Bohm and Hiley, 1975). Thus a specific quantum picture of an intense interaction between remote parts of a system is assumed.

But, as we saw, observed connected with the disjoint subsystems will not detect any strange influence on the results of their measurements and one naturally poses the following question: Is the correlation of the motion of separated subsystems, given by equation (3.1), an expression of an infinite range "quantum interaction" or is it an expression of some simple fact? Let us examine an analogical situation in classical mechanics prior to the detailed discussion of this problem.

Assume that two classical pendulums interact when they are close to each other and do not interact at all after a separation at a larger distance. After the separation every one of the pendulums oscillates freely, but owing to their interaction in the past a specific correlation of their motion exists which will be particularly simple if the pendulums have equal parameters. These correlations are determined by the forces with which the pendulums interact and the way they are separated (and, certainly, the initial conditions of the problem). But the said correlation does not mean that the pendulums influence each other in a magic fashion after their separation. It is only an expression of some property of the simultaneous motion of the two pendulums while any one of them behaves as if it were the only system in the universe (together with a fixed gravitation field).

The wave functions of the type (3.1) have caused a lot of trouble in the past from the point of view of interpretation (especially in the quantum theory of measurement). The above simple example indicates an as simple and natural interpretation: These wave functions describe situations in which the systems x and y are *actually* in states $\psi_i(x)$ and $\varphi_{k_i}(y)$ (with a probability equal to $|a_i|^2$). Owing to the fact, however, that these states are not prepared independently but appear owing to an interaction of the systems x and y in the past, one must use the overall wave function $\phi(x,y)$ for additional information about the correlations in the motion of x and y when this is of interest. An observer of the overall quantum system x, y (analogically to an observer who is interested in the correlations of the motion of some classical systems) thus has the same right to say together with the observers of x and y only that the overall system is really in some state $\psi_i(x)\varphi_{k_i}(y)$ which is encountered with a probability equal to $|a_i|^2$ in a

given ensemble of similarly prepared states. But this observer must not use the usual density matrix calculus for independent events when he is interested in quantities which depend on both x and y after he knows that the two systems have not been independent in the past. The states in which the subsystems actually are can be determined by observer 2 through some act of measurement in which he will not exert any influence on the said states but will only destroy some correlations in the motion of x and y. Thus the only consequence of measurement is that after it not only the subsystems but the overall system too are described by density matrices.

As was mentioned above the complications in the presence of degenerated spectra are inessential, so that they do not need special consideration.

We must remind the reader that the absence of any influence of a remote system on a system of interest is reasonably postulated by conventional quantum mechanics itself (unfortunately this does not seem sufficient for the obtaining of correct numerical results in all cases). Indeed, a remote system will give $U=0$ in the place where the system investigated is located and from equation (2.1) it is evident that in such a case the latter system is "free"—no influence of any kind exists and correlations of motion with other systems play no role in the evolution of the system of interest. We shall not have, certainly, a sensible physical theory if the evolution of a given microsystem depends not only on what takes place inside it but on the behavior, e.g., of some proton in a neighboring town.

Returning to our *Gedankenexperiment* we see that the measurement process, effected by observer 2, does not affect the state of motion of the electron 3 in any way. But this simply means that measurement will give observer 2 information about a process which has taken place "automatically," namely, an irreversible evolution from the initial wave function $\psi_i(\mathbf{r}_3, t)$ given by equation (2.3) to some eigenfunction ψ_f of the new Hamiltonian of electron 3. Measurement thus turns out to play quite a minor role in processes of this type: it just gives information about a fact that has taken place already since nothing is created by measurement except the destruction of some (most often unimportant) correlations, as was mentioned above. Quantum mechanics is thus a quite specific theory. It gives information about all the possible processes and correlations in the system but the price for this is the inability of the theory to describe the concrete relaxation processes in the subsystems which objectively take place (prior to any measurement). And if something takes place objectively, then a theory able to describe it must certainly exist.

In simpler thought situations of the type proposed first by EPR (cf. also the much exploited *Gedankenexperiment* of Bohm, 1952), one has the same reasons to declare that correlations between separated subsystems do

not mean influence. Our discussion of equation (3.2) is applicable in fact (with corresponding modifications) to any case of separated subsystems in which some conservation law for the total system exists. (Bohm's discussion 1952, end of point 17, e.g., of this problem in his specific *Gedankenexperiment* is in fact a private case of ours as far as independence of the conduct of the two systems is concerned; his present-day standpoint on the subject seems to be quite different, though—see Bohm and Hiley, 1975.) In any specific case the role of the said correlations in the motion of separated parts may become important only if they come close together again (without any spurious influence on these correlations by measurement)—the overall system thus formed will have then the same value of the quantity which is constant in the process. In Bohm's example, e.g., we shall have once again a spin-0 molecule. But in the classical case correlations play an exactly analogical role—if several separated subsystems come close together again, then the correlations in their initially "free" motion will play an important role in the subsequent evolution of the total system (they will determine, e.g., the initial conditions in the moment in which interaction is restored). And we have to stress here that one should distinguish between correlations in the case of motion of subsystems of a larger system and the specific quantum interference effects in the case of a *single particle* (leading, for instance, to some diffraction pattern after the passing of a particle flow through a crystal lattice). Indeed, there exist "interference terms" in $|\phi(x,y)|^2$ too but the two things are nevertheless quite different. Lack of clarity in the understanding of this point exactly has led to the establishing of the following conventional standpoint on quantum measurement theory which is postulated to have a general validity:

> Any state of a given subsystem is created by the act of measurement (even when it is carried out on a far-off cite). One must not speak about an actual existence of a fixed state of a part of a larger system prior to measurement since then specific quantum correlation effects will be absent.

But we saw already that this is not the only possible standpoint, and that this is not a reasonable standpoint. The numerous fruitless debates on the role of measurements in the past and the existing paradoxes of the Schrödinger cat type (the cat is neither alive nor dead before the observer kills it or lets it live with his glance) are an evidence of the futility of such a generalized point of view. The point of view of our observer 2 removes the difficulties: after the separation the subsystems of the larger system come "by themselves" to states of definite energies. The experimental verification of this fact will remove some correlations in their motion but this does not mean that measurement has created the states of the subsystems. What

this does mean is, e.g., that Schrödinger's cat will be actually either alive or dead when the observer looks at it to simply verify this fact since superposition of states of different energies of a given system (the cat) are forbidden (cf. also our subsequent discussion).

The results of the discussion in the present section are important and it is worthwhile to summarize them. We saw that (2.2) is an acceptable axiom in quantum mechanics [unlike (2.1)] but the wave functions of the type (2.5) to which it leads need a reinterpretation. They do not give evidence about any specific nonseparability of subsystems. The behavior of a subsystem is completely free of any influence from the remaining subsystems after their separation but it is not independent of their behavior prior to the spurious influence of, e.g., a measurement procedure, owing to the fact that the subsystems have interacted in the past. There is nothing specifically quantal in the latter fact except the statistical character of the correlations in the motion of the subsystems. In classical mechanics we have a completely analogous situation with the exception that when we examine the overall system the corresponding correlations in the motion of subsystems have a deterministic character (in our specific example we can say, for instance, that "the two pendulums oscillate synchronously"). The lack of clarity in understanding that *no influence* and *independence* of the motion of subsystems are two different things (see, e.g., Furry's article, 1936, as an example) lead to assertions of the following type:

> If the separated subsystems are really in definite states determined by their own parameters then one must employ a density matrix and not a WF [wave function] in calculations referring to the overall system. This, however, may lead (and actually leads) to disagreement with experiment, so that the above opinion is erroneous.

What we saw is that one can very well say that both prior to and after a measurement the subsystems are actually in definite states of their own. The only difference between the two cases is that prior to a measurement the overall system is described by a wave function while after a measurement by a density matrix. Or, to put it briefly, lack of influence is sufficient to say that separated subsystems have to be in individual states of their own but independence of the motion of subsystems (leading to an overall density matrix) can be achieved only after interaction with a body extraneous to the overall system of interest. This way of reasoning, being a direct consequence of quantum mechanics itself when it is adequately interpreted, is by no means paradoxical. At the same time it removes old prejudice, leading, for instance, to the explicit or inexplicit conception that if one uses HVs for the physical description of an overall system, then he must use the calculus for independent events as regards its different

subsystems (cf., e.g., Bell's, 1964, argument). Our argument, which says that there exist no specific nonlocality effects for spatially separated subsystems and that lack of influence and independence of their motion are quite different things at the same time, refutes the idea about the necessity of a classical probabilistic calculus in HV theories.

4. NECESSITY OF TIME-IRREVERSIBLE EVOLUTION EQUATIONS FOR SUBSYSTEMS OF A LARGER SYSTEM

Any possible experimentalist in this world is in the position of an observer of a subsystem of a larger system, the latter system being the universe. It is natural to assume that the universe is in a state of definite energy since otherwise it would not be possible to give a clear explanation of the energy conservation law. If this is really so, then any system will undergo a transition to a state of definite energy in the process of its separation from the remaining part of the universe. But the above "universal" assumption is not, strictly speaking, necessary since, as it is postulated in quantum theory (and confirmed by experiment for some simplest cases) any isolated system can be in a state of a definite stationary energy, and in such a case if it splits into several noninteracting parts, then any part will pass into some state of a definite energy (the overall system then plays the role of a "universe"). The observer with his apparatus and the system of interest form, in principle, a "universe" which can be in a state of definite energy. If the system were in a state of definite energy prior to its separation from the rest of the "universe" and well-defined potentials existed during the process of its separation, then the problem for the description of the process of a transition from the (formerly) stationary state ψ_i to a stationary state ψ_f appears for an observer of the system in accord with the above reasoning of observer 2.

The first thing to do for this purpose is to choose the language in which such transitions can be described. In the case of well-defined potentials one could use the basic idea of quantum mechanics of describing the states of motion with the help of a wave function ψ of some sort (this concept has proved to be of a great importance and usefulness for nonclassical phenomena). Then a second problem arises, namely, what is (are) the equation(s) which has (have) to be satisfied by this wave function. Here exactly lies the main difficulty. The discussion of our thought experiment shows that, generally, this will not be the SE (2.1). Then we saw that the transitions to ψ_f are "automatic" and have all the characteristic features of an *irreversible process* (ψ_f is stable and no longer varies with time). The only difference from usual statistical mechanics is that there are

several or even infinitely many "directions" (possibilities) of an irreversible relaxation of the initial wave function in our case. Once the direction is chosen, a definite final result ψ_f will inevitably follow. But how can one choose the direction?

There seem to exist two possible answers. One of them is the following: Owing to the fact that the initial wave function ψ_i is fixed, a set of possible evolution equations exists, leading to different final states ψ_f. The particle makes a random choice of the equation for each concrete transition i–f with a probability P_{if}.

The second possible answer is that the equation of motion is only one, the different final results being determined by additional HVs which have to be known together with ψ_i. (Certainly, things may turn out to be essentially more complex since the HVs may be the fundamental quantities, and the wave function a "macroscopic" property explained by them, e.g., of the type of pressure in thermodynamics.) From the point of view of the transition to the adiabatic results of quantum mechanics which are described by a single equation, the SE, this possibility seems preferable to us. From the above discussion it follows that irreversibility will be a characteristic feature of such a "master equation." Thus we come with help of an independent argumentation to an idea proposed by Bohm and Bub (1966). Our way of reasoning shows a way for finding experimental evidence in favor of the concept that present-day theory is not the last word with respect to the conceptual foundations of quantum mechanics. Another way of searching experimental evidence for this is surveyed, e.g., in the work of Freedman and Holt (1975). We must point out, however, that the latter way (photon correlation experiments) seems somewhat dubious from the point of view of our discussion of the physical meaning of correlations in Section 3. The essential difference in our case is that evidence should be sought in cases of fastly varying (with time) *external potentials*.

It is easily explicable, in principle, why possible HVs will play a noticeable role in the case of swiftly varying perturbations, while in the other limiting case their existence may not be taken into account for reasonable time intervals (for very large time intervals this will not be generally true; cf. T4 and T5). In the case of slow perturbations the HVs may be supposed to have an almost equilibrium distribution which makes the applicability of the SE possible. In the opposite case some non-equilibrium property, for instance, the momentary location of the particles, can play a decisive role in the future evolution of the system. The specific example considered in detail in Appendix B shows that location in regions, not coinciding with the entire region of interest, may become an important parameter in a future theory.

The HVs can turn out to be unobservable in principle. Still, the search for them will not be unreasonable. This opinion is supported in the discussion in T7.

We know practically nothing about the exact form of the hypothetical time-irreversible evolution equation at present. But the application of the SE to the overall system containing the system of interest gives some information about the relaxation properties of the subsystem. Indeed, the interaction of the parts of an overall system is time independent and, consequently, the SE is valid. When, however, the parts of the system move in such a way that up to a certain moment t_0 the interaction between them can be considered to be practically absent and in t_0 instantaneously included, one may hope that the application of the theory of perturbations to the *overall* system can give correct qualitative results about the picture of the time evolution of the said system. This is sufficient for our consideration here, so that one can turn to the discussion in Sections 84 and 112 of Blokhintsev's book (1963). It is shown there that the coefficients with which the wave functions corresponding to energy E participate in the wave packet differ essentially from zero for times $t \lesssim \hbar/|E-E_i|$ only, E_i being the initial energy. For larger t these coefficients practically disappear. If we examine a part of the system which has to "relax" to a state $\psi_n(x)$ corresponding to an eigenenergy E_n when $t \to \infty$, then owing to the completeness of the set of eigenfunctions of the Hamiltonian $H(x)$ the wave function $\psi(x,t)$ of subsystem x will be a superposition of the type

$$\psi(x,t) = \sum_k a_k(t)\psi_k(x) \tag{4.1}$$

in the process of transition. Having in mind the above inequality, the moduli of the coefficients $a_k(t)$ may be expected to be $\sim \exp[-|E_k - E_n|t/\hbar]$, $k \neq n$, since in this case the said inequality will be satisfied. Such $a_k(t)$ would yield an irreversible behavior in time. Certainly, this is not the only form of $a_k(t)$ satisfying $\Delta E \Delta t \sim \hbar$. But the very fact that time irreversibility is intimately related to the well-known time–energy "uncertainty relation" is interesting by itself.

In the above discussion the energy of the other parts of the overall system is considered to be well defined for the sake of simplicity.

Having in mind everything said up to here one may expect that quantum systems obey a master equation of the type

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi + \hat{\Gamma}\psi \tag{4.2}$$

where the operator $\hat{\Gamma}$ becomes significant in the presence of swiftly varying perturbations and contains the possible HVs. Equation (2.2) will reduce to

the usual SE when the last term on its right-hand side is negligible. This equation must be such that an irreversible transition to a state of definite energy be guaranteed in all cases. Detailed considerations of the adiabatic properties of the SE will be given in T4 and T5. It will be shown there that the said term is negligible in all the approximations of perturbation theory only for nonstationary potentials of the type $V(x,t)=V(x)e^{\alpha t}$, $\alpha\to+0$.

In the present paper we have restricted ourselves to the law of energy conservation only because of its exceptional importance in quantum physics—the overall SE is formulated in terms of the energy operator H. A profound study of the role of the other basic physical quantities may probably lead to the elimination of other paradoxes such as the nonexistence of linear combinations of states of the same eigenenergy but different angular momentum in the case of macroscopic bodies.

The argumentation in the present work can well explain an experimental fact. A person interested in the energy state of an atom after the application of some perturbation need not do any measurements despite the prescription that he has to. He has only to wait and see whether the atom will emit a photon in the process of a transition $E_n\to E_m$ $(E_n>E_m)$ or not. The well-defined energy of the photon shows that the atom itself has been in a state of well-defined energy E_n prior to the transition to E_m. It is not necessary thus to try to reduce the initial wave packet of the atom—it will "automatically" reduce itself even if the atom is isolated after the perturbation. A possible proportionality of $|a_k|$ to $\exp[-|E_k-E_n|t|/\hbar]$ gives a good explanation of the fact that the atom quickly reaches a state of energy E_n in the process of self-reduction of ψ, after which a decay owing to the small natural width of energy levels takes place (we are not interested here in the nature of this width).

5. CONCLUSION

Many physicists have come through different ways of reasoning to the thought that preferable variants of the theory exist in which specific difficulties could be absent. A simple objection to all of them was the argument that present-day theory gives correct expressions for the corresponding probabilities and, since nothing more can be obtained experimentally, this is all that we need irrespective of whether one likes the contemporary form of the theory or not.

The discussion in this work aimed for the construction of *Gedankenexperimente* of a different sort than the ones which have extensively been discussed in the literature, and for the demonstration through postulate (2.1) that the above objection is moot. Swiftly varying potentials with time offer a new line of seeking experimental evidence in favor of the

thought that modifications of the theory are necessary. The above discussion of the existing marked disagreement between conventional theory and experiment (Section 2) shows that this line can turn out to be very fruitful. It is most important, obviously, to start with experimental and not only logical argumentation in a difficult field of science and the author hopes that the possibilities which have opened up will not be neglected.

APPENDIX A

The fact that $\psi(\mathbf{r}_3, 0)$ is well defined in our *Gedankenexperiment* at $t=0$ in spite of the existence of an overall wave function for the system of which 3 is a part is fairly obvious by itself. Still, it is worthwhile to show explicitly why this will be really so for systems which consist of a small nucleus and an "electron cloud" about it of a "radius" much larger than that of the nucleus. Taking as an example our specific system and denoting the coordinates of particles 1 and 2 by x we can write the overall Hamiltonian H in the form

$$H = H_x + H_{k3} + V_{x3} \tag{A.1}$$

where H_x is the energy operator of the nuclear particles 1 and 2, including their electromagnetic interaction with each other, H_{k3} the operator of the kinetic energy of 3 and V_{x3} the operator of the electromagnetic interaction of particle 3 with particles 1 and 2. V_{x3} can be represented in the form

$$V_{x3} = V' - 2e^2/r_3 \tag{A.2}$$

where $r_3 = |\mathbf{r}_3|$. V' is essentially not equal to 0 only inside the nucleus, the "radius" $\langle r_2 \rangle$ of which is very small compared to $\langle r_3 \rangle$ owing to the assumed properties of f_{12}. The eigenfunctions ψ_0 of the operator $H_x + H_{k3} - 2e^2/r_3$ corresponding to energy E_0 can, obviously, be represented as

$$\psi_0(x, \mathbf{r}_3) = \varphi_p(x)\psi_q(\mathbf{r}_3) \tag{A.3}$$

where $\varphi_p(x)$ is an eigenfunction of H_x and $\psi_q(\mathbf{r}_3)$ is an eigenfunction of $H_{k3} - 2e^2/r_3$. The operator V' will introduce only a small variation of the values of E_0 and ψ_0 owing to the small dimensions of the nucleus. Indeed, using the theory of stationary perturbations we obtain the following expression for the first-order correction E_0' to E_0:

$$E_0' = \langle \varphi_p \psi_q | V' | \varphi_p \psi_q \rangle = \langle \varphi_p | W_q(x) | \varphi_p \rangle \tag{A.4}$$

where $W_q(x)=\langle\psi_q|V'|\psi_q\rangle$ is small compared to unity owing to the above-mentioned fact. When f_{12} is sufficiently large (A.4) will immediately give $|E_0'|\ll|E_0|$. The same applies to the variation ψ_0' of $\psi_0-|\psi_0|'\ll|\psi_0|$. Thus the wave function of the stationary state of 1, 2, and 3 can be written in the form

$$\psi(x,\mathbf{r}_3)=\varphi_p(x)\left[\psi_q(\mathbf{r}_3)+\phi(x,\mathbf{r}_3)\right] \tag{A.5}$$

where $|\phi(x,\mathbf{r}_3)|\ll|\psi_q(\mathbf{r}_3)|$ for every x. One can say, consequently, that we have really a well-defined initial wave function $\psi_q(\mathbf{r}_3)_{t=0}$ of electron 3. In our specific case this is a direct consequence of the small dimensions of the nucleus.

APPENDIX B

Assume that a particle is moving in an infinitely deep one-dimensional potential well of the form $U(x)=0$ for $0<x<a$, $U(x)=\infty$ for $x\leqslant 0$, $x\geqslant a$. The stationary wave functions, satisfying zero boundary conditions (the necessity of such conditions in our case is discussed in most of the textbooks on quantum mechanics) are of the form $\psi_n(x)=(2/a)^{1/2}\sin(n\pi x/a)$, $n=1, 2,\ldots$, the corresponding eigenenergies being $E_n=n^2\pi^2\hbar^2/2ma^2$. The particle is assumed to be in its first excited state $\psi_2(x)$. Then $\psi_2(a/2)=0$, $\psi_2(a/2+\Delta)=-\psi_2(a/2-\Delta)$, $0\leqslant\Delta\leqslant a/2$.

Let us begin now to insert slowly and symmetrically a "wedge" with absolutely impenetrable walls in point $a/2$. The simple mathematical expression of this action is a "moving" boundary condition of the form $\psi(x)=0$ for $a/2-\delta(t)\leqslant x\leqslant a/2+\delta(t)$, $0\leqslant\delta(t)\leqslant a/2$, $\delta(t)$ being given, e.g., by an expression of the type vt, where v is a small positive constant (we suppose that the edge of the wedge touches point $x=a/2$ in moment $t=0$). Problems of this sort are well known in mathematical physics. In the theory of heat conductivity, for instance, the presence of variable boundary conditions is known as the problem of Stefan.

We shall discuss this problem from the point of view of the SE. The initial wave function $\psi_i=\psi_2(x)$ will, obviously, split into two parts corresponding to the two free regions I and II, separated by a district of length $2\delta(t)$, $t\geqslant 0$, in which $\psi(x,t)=0$. Because of the symmetry of the problem we shall have for any t $\psi[a/2-\Delta'(t),t]=-\psi[a/2+\Delta'(t),t]$, where $\delta(t)\leqslant\Delta'(t)\leqslant a/2$. The evolution of the wave function in any one of the two possible districts in this case of an adiabatic variation of a parameter (here the length of the district) can be found with the help of a specific variant of the theory of time-dependent perturbations (cf., e.g., Davidov, 1973).

Namely, the wave function $\psi(x,t)$ is represented as

$$\psi(x,t)=\sum_n a_{ni}(t)\varphi_n(x,R_t)\exp\left(-i\int_0^t\omega_n(\tau)d\tau\right) \tag{B.1}$$

where $\varphi_n(x,R_t)$ is the eigenfunction of the Hamiltonian corresponding to the value R_t of the valuable parameter R in moment t and $\omega_n(\tau)=\epsilon_n(R_\tau)/\hbar$, $\epsilon_n(R_\tau)$ being the corresponding eigenvalue of H for a value R_τ of the parameter. The initial condition is $a_{ni}(0)=\delta_{ni}$. We shall have then

$$a_{ni}(t)=\int_0^t\phi(\tau)\exp\left(-i\int_0^\tau\omega_{in}(\tau')d\tau'\right)d\tau \tag{B.2}$$

where $\omega_{in}=\omega_i-\omega_n$ and

$$\phi(\tau)=\frac{\dot{R}_\tau\langle n|\partial H/\partial R_\tau|i\rangle}{\epsilon_n(R_\tau)-\epsilon_i(R_\tau)}=-\dot{R}_\tau\langle n|\partial/\partial R_\tau|i\rangle \tag{B.3}$$

These formulas give an expected result—in our adiabatic case the course of the wave function in I and II will practically coincide with the course of the ground state eigenfunction in an infinitely deep potential well of length $a/2-\delta(t)$ for any moment t.

Let us invert now the process in some moment $t>0$, i.e., let us begin to take the wedge adiabatically and symmetrically out. Everything said above remains, obviously, true again. The edge of the wedge will leave point $x=a/2$ in some moment $t=t'$ and the two parts of $\psi(x,t)$ will unite once again. Owing to the symmetry of this adiabatic problem $\psi(x,t)_{t>t'}$ will practically coincide with $\psi_i=\psi_2(x)$, the inessential difference consisting in a phase factor of a modulus equal to unity and some vanishing terms giving the transition probabilities to states $\psi_n(x)$, $n\neq 2$. Thus we shall find our particle in a state of energy E_2 with a probability practically equal to unity.

But this is a paradoxical result. The particle cannot be split into two parts by the wedge. The coherence of the two parts of the wave function in the process of inserting and taking out the wedge is devoid of physical meaning. Indeed, after the edge of the wedge touches point $a/2$ the particle will be located in one of the possible regions of free motion, and after the removal of the wedge the wave function will be a packet of the type

$$\psi(x,t)_{t>t'}=\sum_n a_n\psi_n(x)\exp(-iE_nt/\hbar) \tag{B.4}$$

$\psi_n(x)$ being defined above and the constants a_n being equal to

$\int\psi(x,t')\psi_n^*(x)dx$, $\psi(x,t')\neq 0$ in only one of the intervals $0<x\leqslant a/2$, $a/2<x\leqslant a$. Many a_n thus have to be essentially not equal to 0 according to the same SE for a particle localized in one of the said regions.

One may argue here that only the second way of reasoning is correct, so that the former one must be discarded. But the former argument cannot be discarded since this is the only solution of our problem offered by the SE [the way of reasoning expressed by (B.4) is based on what has to happen and not on what the SE really gives].

Consequently, what we lack in this case is a more detailed information about the location of the particle in moment $t=0$. The necessity of such information is excluded by the formalism of present-day quantum mechanics. So, location can turn out to be an important HV.

The coefficients a_n in (B.4) may not be correct, as our previous discussion shows. This possibility is inessential for the discussion of this specific *Gedankenexperiment*: It is clear from any reasonable point of view that the coordinate distribution has to be essentially nonstationary for $t>t'$, at least for a certain period of time, owing to the possibility for the particle to enter once again the other half of the interval $(0,a)$, while the theory with a variable parameter considered above gives an almost stationary coordinate distribution for any finite period of time.

The above problem was formulated in a somewhat unusual language (variable zero boundary conditions). Its formulation with the help of a *finite* variable potential gives essentially the same results. Such a formulation has the additional merit of demonstrating that no spurious phase shifts of the wave functions in the regions of free motion spring up in result of the sudden appearance or disappearance of a perturbation in point $x=a/2$; in the moments $t=0$ and $t=t'$ we have exactly such a case.

Indeed, let us examine the following finite potential model of the wedge. The touching of point $x=a/2$ is modeled now by an instantaneous appearing of a perturbation $U(x)=0$ for $x\notin(a/2-\delta_0, a/2+\delta_0)$, where δ_0 is a constant, $\delta_0\approx 0$, and $U(x)=U_0=\delta_0^{-2.5}$ for $x\in(a/2-\delta_0, a/2+\delta_0)$, $t\geqslant 0$. Employing the well-known expression

$$D\approx 16\left(\frac{\gamma k_0}{k_0^2+\gamma^2}\right)\exp\left(-\frac{4\delta_0}{\hbar}\left[2m(U_0-\epsilon)\right]^{1/2}\right) \tag{B.5}$$

for the penetrability coefficient D of the barrier where $\gamma=[2m(U_0-\epsilon)]^{1/2}/\hbar$, k_0 is the wave number, and $\epsilon=\hbar^2k_0^2/2m$ is the kinetic energy of a free particle of a mass m, which energy is assumed to coincide with $E_2=4\pi^2\hbar^2/2ma^2$, the energy of the initial state. We see that such a barrier would be practically impenetrable for a free particle of energy E_2 in our potential well.

An easy calculation shows that the SE has the following stationary solutions inside our potential well in the presence of the above barrier U_0: one solution, $\psi_2'(x)$, practically coincides with $\psi_2(x)=(2/a)^{1/2}\sin(2\pi x/a)$ outside $(a/2-\delta_0, a/2+\delta_0)$, $\psi_2'(x)\approx 0$ inside $(a/2-\delta_0, a/2+\delta_0)$ (the same is true for $\psi_2(x)$ inside $(a/2-\delta_0, a/2+\delta_0)$ since $\delta_0\approx 0$); another solution, $\psi_1'(x)$, of practically the same energy E_2 but corresponding to a wave function which is symmetrical with respect to point $x=a/2$ $[\psi_1'(a/2-\Delta)=\psi_1'(a/2+\Delta)]$, and so on. Having this in mind and applying the usual approach to nonstationary problems we find that after the instantaneous appearing of the barrier U_0 the new wave function of the particle will remain essentially equal to $\psi_i=\psi_2(x)$. (The same is confirmed by a straightforward calculation with the help of the theory of nonstationary perturbations, giving practically zero transition probabilities in the first order.)

Let us begin now to expand the barrier very slowly (adiabatically) compared to the characteristic periods corresponding to a typical distance between levels of essentially different energies E_n', keeping its height equal to the initial one ($\delta_0^{-2.5}$) and the symmetry of the problem unchanged (owing to the symmetry of $U(x)$ the wave function $\psi_1'(x)$, corresponding to $E_1'\approx E_2'\approx E_2$, will not play any role since its symmetry with respect to $x=a/2$ is different from the (anti)symmetry of $\psi_2'(x)$, so that despite the presence of quasidegenerated levels the corresponding $\phi(\tau)$ in (B.3) will be equal to zero and the process is thus really adiabatic.) The wave function will remain, obviously, antisymmetric all the time and will practically coincide with the corresponding stationary wave functions in any moment $t>0$. These wave functions, to a good approximation, represent half-periods of sine curves to the left and to the right of the barrier and are approximately equal to 0 inside it. Then we reverse the process as in the wedge case and adiabatically come back to the initial width $2\delta_0$ of the barrier and "switch it off" instantaneously in some moment $t=t'$ (this act corresponds to the removal of the edge of the wedge from point $x=a/2$ at moment $t=t'$). It is obvious that in this way we come essentially back to $\psi_2(x)$ in complete analogy with the former case.

But this physical process has to localize the particle in one of the two possible regions of motion. Indeed, the stationary energy of the particle adiabatically varies in this process owing to its interaction with the expanding barrier. But owing to the impenetrability of the barrier (it can be as impenetrable as one would like if δ_0 is chosen small enough—$U_0\sim\delta_0^{-2.5}$) the particle can interact with one of its walls only. The motion of this wall exactly will cause the variation of the particle's energy. From here immediately follows the same paradox as in the case of an absolutely impenetrable wedge described mathematically by moving zero boundary

conditions. One cannot assert that coherence between parts of a wave function can spring up from not knowing in which one of possible disconnected regions the particle really is if an objective process (exchange of energy) localizing it in one of these regions exists.

It is easy to see that what one needs in fact in order to remove the difficulty is an initial incoherent mixture of two wave functions

$$\psi_{\pm}(x,t)=\frac{1}{2^{1/2}}\left[\psi_1'(x)\exp(-iE_1't/\hbar)\pm\psi_2'(x)\exp(-iE_2't/\hbar)\right] \quad \text{(B.6)}$$

where, as we know, $E_1'\approx E_2'$. Indeed, it is obvious that such linear combinations give a half-period of a sine curve in one of the two regions (say, I) and is practically equal to zero in the other one (say, II). $\psi(x,t)$ will become essentially not equal to 0 in II for a practically infinite period of time in the presence of the barrier; the reader can easily transform our words into specific numbers. The SE and the usual mathematical prescription of working with initial conditions for the wave function cannot transform the initial wave function into the necessary mixture of noncoherent alternatives. What leads to such a mixture is an HV way of reasoning: In moment $t=0$ the particle is either in I or in II and not in I and II at the same time according to current notions. The appearing of the barrier will fix this location without a noticeable influence on the particle's energy. The subsequent removal of the barrier will lead to practically the same picture as the one in the wedge case with noncoherent alternatives.

REFERENCES

Ballentine, L. E. (1970). *Reviews of Modern Physics*, **42**, 358.

Bell, J. S. (1964). *Physics* **1**, 195.

Bell, J. S. (1966). *Reviews of Modern Physics*, **38**, 447.

Blokhintsev, D. I. (1963). *Fundamentals of Quantum Mechanics*. Visshaya shkola, Moscow. (In Russian.)

Bohm, D. (1952). *Quantum Theory*, Chap. 22. Prentice Hall, New York.

Bohm, D., and Bub, J. (1966). *Reviews of Modern Physics*, **38**, 453.

Bohm, D. J., and Hiley, B. J. (1975). *Foundations of Physics*, **5**, 93.

Bohr, N. (1935). *Physical Review*, **48**, 696.

Davidov, A. S. (1973). *Quantum Mechanics*, Section 92. Nauka, Moscow. (In Russian.)

de Broglie, L., Loshak, G., Beswic, J. A., and Vassalo-Pereira, J. (1976). *Foundations of Physics*, **6**, 3.

Einstein, A., Poddolsky, B., and Rosen, N. (1935). *Physical Review*, **47**, 777.

Einstein, A. (1948). *Dialectica*, **11**, 320.

Einstein, A. (1953). *Scientific Papers Presented to Max Born*, p. 33. Oliver & Boyd, Edinburgh.

Feinberg, E. L. (1965). *Yadernaya Fizika*, **1**, 612. (rIn Russian.)

Freedman, S. J., and Holt, R. A. (1975). Comments on Atomic and Molecular Physics, *Comments on Modern Physics: Part D*, **V**, 55.

Furry, W. H. (1936). *Physical Review*, **49**, 393.
Mandel'shtam, L. I. (1950). In *Collected Papers*, Academy of Sciences of the URSS, Vol. V, p. 385. (In Russian.)
von Neumann, J. V. (1932). *Mathematische Grundlagen der Quantenmechanik*, Chap. 4. Springer, Berlin.
Ross-Bonney, A. A. (1975). *Il Nuovo Cimento*, **30B**, 55.
Spighel, M., and Suzor, F. (1962). *Nuclear Physics*, **32**, 346.
Suzor, F. (1960). *Journal of Physics and Radiation*, **21**, 223.
Wigner, E. P. (1963). *American Journal of Physics*, **31**, 6.
Wolniewicz, L. (1965). *Journal of Chemical Physics*, **43**, 1087.